The Creation of a Photon: A Heuristic Calculation of Planck's Constant $\hbar$ or the Fine Structure Constant α

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A dynamical mechanism is presented to calculate Planck's constant $\hbar$, hence α. Because the charge inside the electron performs a zitterbewegung, the relativistic radiation theory is applied to this motion of the charge, rather than to its "center of mass" motion. With a small Doppler-correction the value obtained is $\alpha^{-1} = 137.03$. Some tests of the theory are suggested.

In quantumelectrodynamics (QED) the fine structure constant α enters as a primary input, the coupling constant between electrons and the photon. Creation and annihilation operators are introduced which just create and annihilate photons with a strength given by α. Therefore, within QED there is no way of calculating the value of α, except possibly by self-consistency arguments, when infinitely many perturbation terms are considered.

We wish therefore to go back and consider the process of photon creation or emission in detail. We propose here a dynamical mechanism which allows us to calculate the value of $\hbar$ in a dynamical way. It is based on the radiation theory and, in an essential way, on the Dirac equation for the electron [1]. The basic idea is to apply the radiation formula to the "Zitterbewegung" [2] of the charged particle, and not to the average trajectory of the charge.

The theory thus opens up subphenomena which were summarily lumped into an electron-photon vertex, and has experimental consequences by which it can be tested. It is like looking into this vertex with a magnifying glass.

The "Zitterbewegung" is a rigorous consequence of the Dirac equation. The charge performs, superimposed on the trajectory of the "center of mass" of the system, an "oscillatory motion" around "the center of mass". For the relativistic Dirac particle these notions "center of mass", "oscillatory motion", "zitterbewegung" can all be made mathematically precise. Essentially, one must distinguish between the position operator $\boldsymbol{x}$ that enters the Dirac equation and determines the zitterbewegung of the charge, and the mass-center position operator $\boldsymbol{X}$, of for example Pryce [3], which comes in in the localization problem. Correspondingly, there are two velocities (and two accelerations), the first one being

$$\dot{\boldsymbol{x}} = i[-i\, c\, \boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + \beta\, m\, c^2, \boldsymbol{x}] = c\, \boldsymbol{\alpha}\,, \tag{1}$$

the well-known velocity of the zitterbewegung even in the presence of fields. Hence

$$\dot{\boldsymbol{x}}^2 = c^2\, \boldsymbol{\alpha}^2 = 3c^2\,. \tag{2}$$

There is another intuitive picture of this intrinsic structure of the electron. From a comparison of the dynamical group structure of the Dirac equation [i.e. the $0(4,2)$-dynamical algebra of the Dirac matrices γ_μ] with the corresponding structure of an H-atom (also having a dynamical algebra $0(4,2)$ of matrices Γ_μ) [4], it follows that within an electron the charge performs a Kepler-type motion around a center while the whole "atom" is moving. The difference is that the "internal space" is a discrete space consisting of 4-points, rather than the 3-dimensional continuum of the relative coordinate $\boldsymbol{r}$ of the H-atom. Otherwise the mathematics is exactly the same, and one can exhibit the relative coordinates and internal Hamiltonian as matrices [5]. We can then evaluate average values of all quantities. For example, it is well-known that [6] the spin magnetic moment of the electron can be obtained as the average of the orbital magnetic moment of a charge performing the zitterbewegung. This applies to all other properties of the Kepler motion [5].

Because the electron is not a point particle in the sense described above, we apply the radiation formula to the actual motion (i.e. zitterbewegung) of the charge, *when the electron as a whole is accelerated*; the free electron does not radiate.

In the center of mass of the electron, the invariant radiated energy per unit time is given by

$$P = \frac{2}{3}\, \frac{e^2}{4\pi\, \varepsilon_0\, c^3}\, 4\pi\, \overline{(\ddot{z}_\mu\, \ddot{z}^\mu)}\,, \tag{3}$$

where $\ddot{z}_\mu$ is the acceleration of the zitterbewegung. The factor (4π) in the numerator comes from integrating over all angles of the direction of the zittervelocity. To distinguish from this, we denote by v_μ and $\dot{v}_\mu$, the velocity and acceleration of the center of mass of the electron, respectively.

We consider now a Fourier component of the velocity $\dot{z}_\mu$ of frequency ω and amplitude square

$3c^2$, Equation (2). As is well-known, there is no contradiction with relativity [7] because one always takes expectation values, and one is dealing with a discrete quantum mechanics. The acceleration of this Fourier component has therefore an amplitude squared equal to $\overline{(\ddot{z}_\mu \ddot{z}^\mu)} = 3c^2\omega^2$. The zitter-acceleration has constant magnitude, but variable phase [7]. Thus

$$P = (e^2/4\pi\,\varepsilon_0\,c)\,8\pi^2\,\omega^2\,. \tag{4}$$

The energy radiated during a period $\tau = 2\pi/\omega$ in the proper frame of the oscillating charge, or in $\tau' = (2\pi/\omega)\sqrt{1-\beta_z^2}$ in the center of mass of the electron, is $E = P\tau'$. (Note that P is a scalar and E the zero component of a 4-vector). Thus

$$E = \left[\frac{e^2}{4\pi\,\varepsilon_0\,c}\,16\pi^2\sqrt{1-\beta_z{}^2}\right]\omega \equiv \hbar_0\,\omega\,, \tag{5}$$

or

$$E = \frac{e^2}{4\pi\,\varepsilon_0\,c}\,8\pi^2\sqrt{3}\,\omega \equiv \hbar_0\,\omega\,. \tag{5'}$$

The square of the zittervelocity changes between 0 and $\dot{z}^2_{\max}$; hence the average value of $\beta_z{}^2$ is $(1/2)^2$, or $\tau' = \dfrac{2\pi}{\omega}\,\dfrac{\sqrt{3}}{2}$.

This is the value $\hbar_0$ calculated in the "center of mass" of the electron. However, if this radiated energy is associated with the frequency in the laboratory frame we have to correct the frequency ω by the Doppler effect due to the center of mass velocity $\boldsymbol{v}$, making an angle θ relative to the propagation vector k. This is similar to the Doppler correction of frequencies for moving atoms; after all the model of the electron we are using is an atomic-type model. The connection between the measured frequency and the actual energy transition inside the electron is therefore given by

$$E = \frac{e^2}{4\pi\,\varepsilon_0\,c}\,16\pi^2\sqrt{1-\overline{\beta_z{}^2}}\,\frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta}\,\omega \equiv \hbar\,\omega\,. \tag{6}$$

Consequently, using the value $\sqrt{1-\overline{\beta_z{}^2}} = \sqrt{3}/2$, we have

$$\alpha^{-1} \equiv \left(\frac{e^2}{4\pi\,\varepsilon_0\,\hbar\,c}\right)^{-1} = 8\pi^2\sqrt{3}\,\frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta} = 136.7572\sqrt{\frac{1+\beta}{1-\beta}}\,(\theta = 0)\,. \tag{7}$$

For atomic phenomena, taking $\beta = 2\times 10^{-3}$ ($v = 6\times 10^5$ m/sec), $\sqrt{1+\beta/1-\beta} = 1.002$, we find $\alpha^{-1} = 137.03$ which is the accepted value of the fine structure constant. Without the Doppler correction the initial value of α^{-1} is 136.7572.

The variation of $\hbar$ or α expressed in the relativistic factors in (6) or (7) is surprising, because standard quantum theory accepts α as absolutely constant. On the other hand, this fact may provide a test for the present theory. It implies a kind of level broadening to the sharp formula $E = \hbar\omega$. Most accurate determinations of $\hbar$ or α are indirect [8], based on the Josephson effect or atomic level splittings. In fact the determination of various fundamental constants is quite intermingled with each other. Whether the observed spread in the measured values of α is real, remains to be seen. We suggest that the energy of the photon of a definite frequency should be measured directly as a function of the velocity and acceleration of its source.

We emphasize that the quantization of energy (Planck) comes about because the external interaction H only changes the phase of the zitter-acceleration and not its magnitude:

$$\dot{\boldsymbol{\alpha}}(t) = \exp[2i\int H\,\mathrm{d}t]\,\dot{\boldsymbol{\alpha}}(0)$$

{this follows from Eq. (1) and the equation $\dot{\boldsymbol{\alpha}} = i[H, \boldsymbol{\alpha}]$}. Consequently it is possible in principle to calculate the frequency spectrum of radiation as well, which will be a test of the theory.

There are mathematical formulas and considerable numerology about the value of α [9]. These values are quite different in form from ours. Due to the dynamical nature of the present theory we do not expect an absolute value for α, yet the agreement obtained is remarkable.

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